A METHOD FOR DETERMINING THE VIRTUAL MASS DISTRIBUTION AROUND A VIBRATING SHIP

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Washington, D.C. 20007

A METHOD FOR DETERMINING THE VIRTUAL MASS DISTRIBUTION AROUND A VIBRATING SHIP

by

A.N. Hicks

DEPARTMENT OF STRUCTURAL MECHANICS

RESEARCH AND DEVELOPMENT REPORT

January 1970

Report 3272

37

The Naval Ship Research and Development Center is a U.S. Navy center for laboratory effort directed at achieving improved sea and air vehicles. It was formed in March 1967 by merging the David Taylor Model Basin at Carderock, Maryland and the Marine Engineering Laboratory at Annapolis, Maryland. The Mine Defense Laboratory, Panama City, Florida became part of the Center in November 1967.

Naval Ship Research and Development Center
Washington, D.C. 20007

DEPARTMENT OF THE NAVY
NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER
WASHINGTON, D. C. 20007

A METHOD FOR DETERMINING THE VIRTUAL MASS DISTRIBUTION AROUND A VIBRATING SHIP

by

A.N. Hicks

January 1970 Report 3272

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

A method is presented for determining the three-dimensional virtual mass distribution associated with the vertical girder vibration of ships. The method was developed for use with a lumped mass/weightless beam ship representation and is based on a set of dipole distributions along the ship axis. It provides a virtual mass matrix with off-diagonal elements and enables all the vibration frequencies and shapes of the ship to be computed from a single matrix equation. The usual method for determining the frequencies and shapes uses a separate mass matrix for each mode. The method is preferable to the standard one for short or unusual ships and mode shapes, or where it is desirable to include all modes in a single equation. However, if separate consideration of each mode is acceptable, the standard technique is simpler for normal ships.

ADMINISTRATIVE INFORMATION

The author of this report is a Senior Scientific Officer of the Naval Construction Research Establishment at Dunfermline, Fife, Scotland. During 1969, he was assigned duties as exchange scientist in the Ship Protection Division of the Department of Structural Mechanics, Naval Ship Research and Development Center; his salary and expenses during this assignment were paid by the United Kingdom. The work reported herein was performed under Naval Ordnance Systems Command Task UF17-354-304, with funding support only for computer time and printing of the report.

INTRODUCTION

In problems concerned with the vibration of ships, it has long been appreciated that the effect of the surrounding water must be allowed for if reasonably accurate predictions are to be made of the frequencies of vibration and of the mode shapes. At almost all frequencies of interest, the principal effect of the water is a very large increase in the effective inertial mass of the ship; at heaving but at pitching frequencies, there is also a considerable damping effect due to the generation of surface waves. For many ships, the additional mass (virtual mass) due to the water is between 1 1/2 and twice the total displacement of the ship and its effect

is consequently very significant. Todd[1] gives an historical survey of the study of virtual mass and its application to ship vibration. Kaplan[2] gives a comprehensive critical review of the literature of the subject.

Most current methods of allowing for the virtual mass are adaptations of a technique introduced very early by Lewis.[3] He noted that because of the long slender nature of ships, the fluid flow around the ship would be largely confined between transverse planes because of its transverse motion and so could be approximated at each ship cross section by the two-dimensional flow around an infinite cylinder of the same shape as the cross section. Such two-dimensional flows are fairly easy to determine. The method has since been called strip theory. To allow for the existence of some flow parallel to the ship axis, Lewis computed the exact kinetic energy for vibrating ellipsoids of revolution and also the kinetic energies for the same ellipsoids of revolution under the assumptions of strip theory. For any particular ship, the virtual mass distribution along the ship deduced from the two-dimensional flow solutions could then be reduced in the ratio of the two kinetic energies calculated for an ellipsoid of the same length/beam ratio. This is still the standard method of computing the virtual mass distribution and the two-dimensional flows have been calculated for a much greater variety of cross-sectional shapes than were given by Lewis. However, Taylor[4] showed that Lewis had not used the best possible boundary condition for his exact ellipsoid solution. Taylor presented a different set of reduction factors to allow for the three-dimensional flow effect. He also gave a third set of reduction factors deduced from the transverse vibration of an infinite circular cylinder with a sinusoidal distribution in the vibration amplitude along its length. Most vibration calculations have used his ellipsoid correction factor but at least one method[5] uses the cylinder factor.

The above technique works very successfully for the basic two or three vibration modes, but it has several disadvantages. First, in addition to the variety of three-dimensional flow correction factors

[1]References are listed on page 30.

available, each type of reduction factor depends on the particular type of motion; a different factor is required for each vibration mode. Lewis gave correction factors for heaving, pitching, and two- and three-node vertical vibration. Taylor gave a correction factor only for two-node vibration. Since most methods of computing the vibration frequencies involve eigenvalues of either a differential equation or a matrix, this means that a different equation or matrix must be solved for each mode; this requirement considerably complicates programming a computer to carry out the task. Moreover, for some shapes (as discussed at some length in Reference 2), there is the possibility that the correction factor may vary along the length of the ship, the reduction possibly being greater near the ends than at the center. There is no present means of allowing for such a variation. Finally, in one particular type of problem, namely the hull whipping induced by underwater explosions, in order to be able to deal with nonlinear effects conveniently it is necessary to represent the equations of motion of the ship as a single matrix equation including the effects of all modes simultaneously. The present type of correction factor, varying from mode to mode, excludes such a representation. Even in the completely linear case, the vibrations of a damaged ship involve mode shapes for which no correction factors are available. These disadvantages prompted the present attempt to find an alternative method of allowing for the virtual mass effect.

NATURE OF THE MODIFICATION REQUIRED

The present "strip theory" described above relates the force on each transverse section of the ship to the motion of that section alone, the overall (three-dimensional) correction factor being constant along the length. If the ship is divided into n sections, with n large enough that the variation in cross-sectional shape is small in each section, then the vertical hydrodynamic force F_{wi} on the i^{th} section is related to the average vertical displacement y_i of the section by

$$F_{wi} = - m_{wi} \ddot{y}_i$$

where dots denote differentiation with respect to time and m_{wi} is the virtual mass of the section. The equation for the forces on all sections

of the ship may then be written in the matrix form

$$\underset{\sim}{F}_w = - M_w \ddot{\underset{\sim}{y}} \qquad [1]$$

where $\underset{\sim}{F}_w$ and $\underset{\sim}{y}$ are column vectors and M_w is a diagonal matrix. This standard strip theory equation clearly cannot be valid for general motions. If one section of the ship is accelerating upward, it will certainly experience a downward hydrodynamic force and so diagonal elements in M_w are necessary. However it will also induce a downward fluid acceleration around it as fluid moves to fill the space it is vacating. This flow will be around neighboring sections and so will induce a downward force on these too. This means that for Equation [1] to be true, M_w should contain off-diagonal elements. In fact, all elements of M_w will be nonzero, but since each section will principally affect its nearest neighbors, the magnitude of the elements will decay rapidly away from the main diagonal. The problem is to determine the elements of M_w. Since the general ship problem is very difficult, attention is directed first to the case of an axisymmetric ship.

MATHEMATICAL MODEL FOR AN AXISYMMETRIC SHIP

For such a ship, it is possible to satisfy fairly well the fluid boundary condition of equality of hull and fluid velocities along the normals to the hull by means of a distribution of vertical dipoles distributed along the axis of the ship. This distribution should normally be continuous, but if the ship is considered to be divided into a number of sections and the number is large enough, then it should be reasonable to assume that the line distribution in each section i has a constant strength μ_i.

Since the velocity potential at the point (r,θ,z) due to a dipole at the point (o,o,s) (referred to the cylindrical polar coordinates of Figure 1) is given by

$$\phi = \frac{\mu \; r \cos \theta}{[r^2+(z-s)^2]^{3/2}} \; , \text{ where } u = - \nabla\phi \; ,$$

the potential due to a line distribution of dipoles of strength μ_i per unit length and extending from $z_{i-1/2}$ to $z_{i+1/2}$ is

$$\phi_i(r,\theta,z) = \mu_i \; r \cos\theta \int_{z_{i-1/2}}^{z_{i+1/2}} \frac{ds}{[r^2+(z-s)^2]^{3/2}}$$

whence

$$\phi_i = \frac{\mu_i \cos\theta}{r} \left\{ \frac{z-z_{i-1/2}}{[r^2+(z-z_{i-1/2})^2]^{1/2}} - \frac{z-z_{i+1/2}}{[r^2+(z-z_{i+1/2})^2]^{1/2}} \right\} \qquad [2]$$

The radial velocity u_r produced by the distribution is given by

$$u_{ri}(z) = -\frac{\partial\phi_i}{\partial r} = \frac{\mu_i \cos\theta}{r^2} \left\{ \frac{z-z_{i-1/2}}{[r^2+(z-z_{i-1/2})]^{1/2}} - \frac{z-z_{i+1/2}}{[r^2+(z-z_{i+1/2})^2]^{1/2}} \right.$$

$$\left. + \frac{r^2(z-z_{i-1/2})}{[r^2+(z-z_{i-1/2})^2]^{3/2}} - \frac{r^2(z-z_{i+1/2})}{[r^2+(z-z_{i+1/2})^2]^{3/2}} \right\} \qquad [3]$$

and the longitudinal velocity u_z by

$$u_{zi}(z) = -\frac{\partial\phi_j}{\partial z} = \mu_i \; r \cos\theta \left\{ \frac{1}{[r^2+(z-z_{i+1/2})^2]^{3/2}} - \frac{1}{[r^2+(z-z_{i-1/2})^2]^{3/2}} \right\} \qquad [4]$$

The total velocities produced by all the distributions are therefore

$$u_r(z) = \sum_{j=1}^{n} u_{rj}(z) \qquad \text{and} \qquad u_z(z) = \sum_{j=1}^{n} u_{zj}(z)$$

The boundary condition at the hull is that the velocities of the fluid and the hull, resolved in the direction of the normal to the hull,

should be equal. For the axisymmetric shape considered, the angle λ between the normal to the hull and the radial direction is given by $\lambda = - \tan^{-1} b'$ where $b(z)$ is the radius of the ship and $b' = db/dz$. The boundary condition is, therefore,

$$v \cos \lambda - v' b \sin \lambda = u_r \cos \lambda + u_z \sin \lambda \qquad [5]$$

where $v(z)$ is the distribution of vertical velocity along the length. This condition allows for both shearing and flexing of the ship. For the more interesting, lower frequency modes of vibration, $v'b$ will be small compared to v since the wavelength will be much greater than the half beam. λ will also normally be small except possibly in the immediate neighborhood of the stern. The term $v'b \sin \lambda$ will therefore be very small. The flow along the ship, (u_z) is produced partly by the variation in v along the ship (i.e., by v') and partly by the changing shape of the ship (b'). It too will, therefore, normally be small when v' and b' are small. It is generally possible then to simplify the boundary condition (Equation [5]) to

$$v = u_r \qquad [6]$$

except when rapid changes occur in either the ship underwater cross-sectional area or in the velocity distribution along its length (e.g., for high modes). This simplification in the boundary condition slightly reduces the amount of data needed to specify the ship and its motion. It is roughly equivalent to neglecting rotary inertia in the dynamic equations of the ship itself.

Neither of the boundary conditions, Equations [5] and [6], can be satisfied everywhere by the assumed velocity potential. However, either can be satisfied at up to n "collocation" points along the ship and the boundary condition will not be seriously violated anywhere if these points are suitably chosen. The most convenient choice for the collocation points is at the midpoints z_i of the ship sections, and satisfying Equation [6] at these points gives

$$\dot{\underset{\sim}{y}} = \frac{1}{b^2} A \underset{\sim}{\mu} \qquad [7]$$

Here b is the maximum value of $b(z)$, $\dot{y}(z)$ is the vertical velocity of the ship, and the matrix $A = (\alpha_{ij})$ where

$$\alpha_{ij} = \frac{b^2}{b_i^2}\left\{\frac{(z_i - z_j + \ell_j/2)}{[b_i^2 + (z_i - z_j + \ell_j/2)^2]^{1/2}} - \frac{(z_i - z_j - \ell_j/2)}{[b_i^2 + (z_i - z_j - \ell_j/2)^2]^{1/2}}\right.$$

$$\left. + \frac{b_i^2 (z_i - z_j + \ell_j/2)}{[b_i^2 + (z_i - z_j + \ell_j/2)^2]^{3/2}} - \frac{b_i^2 (z_i - z_j - \ell_j/2)}{[b_i^2 + (z_i - z_j - \ell_j/2)^2]^{3/2}}\right\} \qquad [8]$$

Equation [7] may then be inverted to determine the strengths $\underset{\sim}{\mu}$ of the line distributions necessary to satisfy the boundary condition, Equation [6], for the given velocity distribution $\underset{\sim}{\dot{y}}$. This gives

$$\underset{\sim}{\mu} = b^2\, A^{-1}\, \underset{\sim}{\dot{y}}$$

The upward vertical force per unit length on the section at the point z_i is given by

$$f_i = -2 \int_0^{\pi} p \cos\theta\, b_i\, d\theta$$

where $p = \rho\dot{\phi}$ is the fluid pressure. Thus

$$f_i = -2\rho \int_0^{\pi} b_i\, \dot{\phi} \cos\theta\, d\theta = -2\rho \left\{\frac{b}{\ell_i} \sum_{j=1}^{n} \beta_{ij}\, \dot{\mu}_j \int_0^{\pi} \cos^2\theta\, d\theta\right\}$$

$$= -\frac{\pi\rho b}{\ell_i} \sum_{j=1}^{n} \beta_{ij}\, \dot{\mu}_j$$

where

$$\beta_{ij} = \frac{\ell_i}{b}\left\{\frac{z_i - z_j + \ell_j/2)^2}{[b_i^2 + (z_i - z_j + \ell_j/2)^2]^{1/2}} - \frac{z_i - z_j - \ell_j/2)}{[b_i^2 + (z_i - z_j - \ell_j/2)^2]^{1/2}}\right\} \qquad [9]$$

Since in any actual motion, conditions along the ship will be continuous, the upward force/unit length at the center of the section will approximate the average force/unit length over the whole section and the total upward force F_i on the section will be approximately

$$F_i = \ell_i f_i = -\pi\rho b \sum_{j=1}^{n} \beta_{ij} \dot{\mu}_j$$

that is

$$\underset{\sim}{F} = -\pi\rho b B \dot{\underset{\sim}{\mu}} = -\pi b^3 \rho\, B\, A^{-1} \ddot{\underset{\sim}{y}} \qquad [10]$$

where B is the matrix (β_{ij}). The required inertial water mass matrix M_W is therefore given by

$$M_W = \pi b^3 \rho\, B\, A^{-1} \qquad [11]$$

This matrix depends only on the shape of the ship and is completely independent of the type of motion (or vibration mode). With M_W known, the force distribution for any vertical acceleration distribution $\ddot{\underset{\sim}{y}}$ is readily found. A short computer routine has been written to compute the non-dimensional matrix BA^{-1} and the force distributions deduced from it, by Equation [10], for given distributions of vertical acceleration.

COMPARISON OF RESULTS WITH KNOWN EXACT FORCE DISTRIBUTIONS

Exact solutions are known for two forms of fluid flow that are suitable for comparison, namely, the flows around a vibrating prolate spheroid and around an infinite circular cylinder whose transverse velocity varies sinusoidally along its length. The exact results for both cases have been compared with the results from the foregoing analysis.

VIBRATING PROLATE SPHEROID

The methods involved in the solution of the flow about a prolate spheroid are discussed in some detail by Lamb, (see page 139 of Reference 6). Lewis[3] gave the first solution in connection with transverse shear vibrations and Taylor[4] gave a second solution using a different, more realistic boundary condition involving both flexure and shear. This type of motion has also since been investigated by Landweber and Macagno.[7]

In principle, the vibrating ellipsoid can be solved exactly for any arbitrary transverse velocity distribution but in practice only distributions represented by low order polynomials are required. Using the analysis outlined in the Appendix, a short computer routine was written

to evaluate the force distribution on an ellipsoid with a velocity distribution representable by

$$v(z) = \sum_{n=1}^{5} v_n (z/a)^{n-1}$$

where z is the distance along the axis of symmetry from the center of the ellipsoid of length 2a (see Figure 2). This velocity distribution is sufficient to approximate heaving, pitching, and the first three whipping modes of ship vibration. The boundary condition used in the solution is that of Taylor and allows for flexure as well as shear.

Figures 3 and 4 compare the transverse force distributions given by the approximate analysis with the values given by the exact solution and also with the values given by the strip method, using the Lewis correction factors since these are available for four of the five modes. The values used for the coefficients $(v_1,\ldots,v_5)$ for these cases are given in Table 1.

TABLE 1

Coefficients for the Ellipsoid Vibration Shapes

Mode \ Coefficient	V_1	V_2	V_3	V_4	V_5
Heave	1	0	0	0	0
Pitch	0	1	0	0	0
2-node Vertical	-0.200	0	1	0	0
3-node Vertical	0	-0.429	0	1	0
4-node Vertical	0.0274	0	-0.534	0	1

The first four mode shapes are those used by Lewis, although the exact analysis used the better Taylor boundary condition. The Lewis correction factor and the approximate three-dimensional analysis both use the shear type boundary condition, and the three-dimensional analysis also assumes that the rate of change of the radius along the length is small. The fifth mode shape has been chosen to have nodes at 0.155L, 0.38L, 0.62L,

and 0.845L, where L is the total length. These are about the correct positions for destroyers, but the resulting shape gives rather too much prominence to the ends and too little to the central section.

For the L/B=10 ellipsoid, the results for both the three-dimensional and the two-dimensional (strip theory) approximations agree well with the exact analysis and there is little to choose between them. The three-dimensional approximation is slightly better near the center of the ellipsoid where changes in the radius are smallest, but the two-dimensional approximation is better at the ends where the radius is changing rapidly.

The results for the L/B=5 ellipsoid are very similar but the divergence from the exact solution is quite serious near the ends for both approximations for modes as low as the second vibration mode. Once again, there is very little to choose between the two approximate methods.

For most ships in which vibration frequencies are particularly important, the L/B ratio is near 10 and both the strip theory and the new three-dimensional approximation should give good results. The divergence near the ends for the lower L/B ratio is caused by using the approximate form of the boundary condition, Equation [6]. From this point of view, the ellipsoid is rather a poor shape since the radius changes extremely rapidly near the ends. The radius changes are much less severe for typical ship forms, and either approximate method would give better results. Use of the exact boundary condition, Equation [5], would be simple in the case of the ellipsoid because the velocity distribution and rate of change of radius are easily defined, but is scarcely worth the effort for ship forms (unless rotary inertias are being considered). This point is considered later.

INFINITE CIRCULAR CYLINDER

Taylor[4] was the first to consider this case. The infinitely long circular cylinder was assumed to have a transverse velocity distribution

$$v(z) = v_0 \cos kz$$

Taylor gave the velocity potential for this distribution as

$$\phi = \frac{v_o \, K_1(kr)}{k \, K_1'(kb)} \cos\theta \cos kz$$

where b is the cylinder radius and K_1 is a modified Bessel function of the second kind. The force distribution may be found as before and is

$$f(z) = -2 \int_0^{\pi} p \cos\theta \, b \, d\theta = -2\rho b \int_0^{\pi} \dot{\phi} \cos\theta \, d\theta = -\pi\rho b^2 \frac{\dot{v}_o \, K_1(kb)}{kb \, K_1'(kb)} \cos kz$$

(force/unit length)

The wavelength λ of the velocity distribution is $\lambda = 2\pi/k$.

Since the approximate analysis is based on a body of finite length, it cannot give a uniformly good representation of the infinite cylinder. However, if it is used to represent three complete wavelengths of the cylinder, the flow in the central wavelength should be approximately correct. With three wavelengths, the program restricts the number of sections in each wavelength to six. Table 2 compares the results from the approximate analysis with the exact results. In this case, the cylinder is of uniform diameter so that the boundary conditions, Equations [5] and [6], in the three-dimensional approximation are equivalent. Inaccuracies in the solution are due either to the coarseness of the representation or to the finite length of the cylinder in the three-dimensional approximation.

TABLE 2

Values of $f(z)/\pi\rho b^2\dot{v}_o$ for an Infinite Cylinder

λ/b \ z/λ		$\pi/6$	$\pi/2$	$5\pi/6$	$7\pi/6$	$3\pi/2$	$11\pi/6$	$13\pi/6$	$5\pi/2$	$17\pi/6$
5	exact	-0.448	0	0.448	0.448	0	-0.448	-0.448	0	0.448
	3-D	-0.445	-0.0002	0.445	0.445	-0.0014	-0.449	-0.452	-0.0140	0.383
10	exact	-0.631	0	0.631	0.631	0	-0.631	-0.631	0	0.691
	3-D	-0.609	-0.0003	0.609	0.609	-0.0013	-0.611	-0.616	-0.0189	0.511
15	exact	-0.762	0	0.762	0.762	0	-0.762	-0.762	0	0.762
	3-D	-0.741	-0.0001	0.741	0.741	-0.0004	-0.744	-0.745	-0.0096	0.676
20	exact	-0.826	0	0.826	0.826	0	-0.826	-0.826	0	0.826
	3-D	-0.839	-0.0007	0.839	0.839	0.0007	-0.839	-0.840	-0.0015	0.823

For all values of λ/b, the forces are clearly very accurate near the center, being within 4 percent of the exact value in all cases. The points given were the only ones used, indicating the coarseness of the representation; it is equivalent in a ship representation to using 20 points to represent the fifth vibration mode (six nodes). Since the values from the three-dimensional approximation are constant to at least $z/\lambda = 7\pi/6$, the differences from the exact result are attributable to the coarse mesh rather than to the finite length. For the $\lambda/b = 5$ case, the lengths of each section are nearly equal to their radii whereas for the $\lambda/b = 20$ case, the sections are nearly nine radii long.

APPLICATION OF THE TECHNIQUE TO NONAXISYMMETRIC SHIPS

Clearly it would be possible in principle to extend the three-dimensional approximation by adding distributions of higher multipoles along the ship axis and determining their strength by satisfying a boundary condition around the circumference of the ship as well as along its axis. This would, however, require very large amounts of data to represent the ship shape as well as the inversion of a very large matrix. The success of the strip method using Lewis sections for the cross-sectional shapes points to simpler approaches.

In the solution of the two-dimensional flows about ship-type cross sections, although the velocity potential may in fact consist of a superposition of two-dimensional multipoles of all orders, the added mass of each section depends only on the dipole term, the shape of the section determining its strength. In the three-dimensional case, it should, therefore, be approximately correct to account for the shape of the section via the strengths of the dipole distributions.

Since the original work by Lewis, it has been customary to represent the added mass per unit length of the two-dimensional cross sections in the form

$$\frac{\pi}{2} \rho\, b^2\, C$$

where b is the half beam of the section and C is a constant depending on the section shape. Values for C have been computed for a great variety of

shapes.[3,4,8-12] The above value for the added mass is also that due to a circular cylinder of radius

$$b_{equiv} = b\sqrt{C} \qquad [12]$$

For a nonaxisymmetric ship then, each cross section may be compared with the known shapes and C determined. Then Equation [12] gives the appropriate radius for an axisymmetric approximation to the actual shape. There are no reasonably simple nonaxisymmetric three-dimensional flows with exact solutions which can be used for comparison, but the procedure should give reasonable results. Certainly in the 2-node vibration mode, in the central section of the ship where the added mass is most important, the technique will give very good answers since the flow in this region is very nearly two-dimensional and the method is exact in the two-dimensional case.

APPLICATION TO SHIP VIBRATION

Most current techniques[5,13] for the determination of ship natural frequencies by purely theoretical means depend on finite-element lumped mass approaches. These represent the ship as a series of lumped masses interconnected by weightless elastic beams. All applied forces, including distributed inertial forces and moments, are approximated by equivalent point forces and moments applied to the lumped masses. It is then possible to compute a stiffness matrix K such that when no moments are applied, the forces $\underset{\sim}{F}$ required at the masses to statically maintain a given displacement shape $\underset{\sim}{y}$ are given by

$$\underset{\sim}{F} = K \underset{\sim}{y}$$

Neglecting buoyancy forces and rotary inertia (these can easily be included if desired), the only forces on a ship in still water are inertial forces $\underset{\sim}{F}_I$ and hydrodynamic forces $\underset{\sim}{F}_W$. If the values of the lumped masses are $m_1, \ldots, m_n$

$$\underset{\sim}{F}_I = - M \ddot{\underset{\sim}{y}}$$

where M is the diagonal matrix with elements (m_i). The hydrodynamic forces are given by Equations [10], i.e.,

$$\underset{\sim}{F}_w = - M_w \ddot{\underset{\sim}{y}}$$

with M_w as given in Equation [11].
The equation of ship motion is then

$$K \underset{\sim}{y} = \underset{\sim}{F} = - M\ddot{\underset{\sim}{y}} - M_w \ddot{\underset{\sim}{y}} \qquad [13]$$

i.e.
$$(M + M_w)\, \ddot{\underset{\sim}{y}} + K \underset{\sim}{y} = 0$$

The natural vibration frequencies are the eigenvalues of this matrix equation and the mode shapes are the corresponding vectors. In the strip method for the hydrodynamic flow, M and M_w are both diagonal matrices. Since K is symmetric, the equation can then be transformed into

$$\ddot{\underset{\sim}{z}} + S \underset{\sim}{z} = 0$$

where $S = (M + M_w)^{-1/2}\, K\, (M + M_w)^{-1/2}$ and $\underset{\sim}{z} = (M + M_w)^{1/2}\, \underset{\sim}{y}$.
S is symmetric and its eigenvalues are easily found by any of the standard routines for eigenvalues and vectors of symmetric matrixes. For the strip method, however, M_w depends on the mode shape being investigated and a different matrix S must be used for each mode.

In the proposed three-dimensional analysis, the matrix M_w is found as a full matrix with a dominant diagonal but no zero elements. Equation [13] may then be written

$$\ddot{\underset{\sim}{y}} + S_1 \underset{\sim}{y} = 0 \quad ; \quad S_1 = (M + M_w)^{-1} K$$

and the eigenvalues found directly. S_1 will not, however, be symmetric, and this restricts the available range of computer routines. The full M_w matrix is actually not symmetric but the degree of asymmetry is not large except for extreme shapes. It may be artifically made symmetric by replacing all elements m_{wij} by $1/2\ (m_{wij} + m_{wji})$. This procedure was carried out for the examples used to check the three-dimensional theory and in no case did it change the resulting forces by more than 3 percent. Since the vibration frequencies depend, approximately, on the square root of the

mass, this difference is negligible. A standard Choleski decomposition routine[14] may then be used to generate a matrix L such that

$$LL^T = (M + M_w)$$

Equation [13] then transforms into

$$\ddot{\underset{\sim}{z}} + S_2 \underset{\sim}{z} = 0$$

where $S_2 = L^{-1} K L^{-T}$. S_2 is now symmetric and its eigenvalues and vectors may again be found by standard symmetric routines.

COMPARISON WITH FULL-SCALE SHIP VIBRATION RESULTS

Frequencies and mode shapes for overall hull vibrations were measured recently on a World War II 2500-ton destroyer, HMS ROEBUCK. Table 3 compares measured frequencies with those calculated using both standard strip theory and the three-dimensional flow approximation. The strip theory results were obtained by using Lewis three-dimensional correction factors since these are available for heave, pitch, and the first two vibration modes and could be estimated for the third and fourth modes by extrapolation. Also included in the table are the results calculated for the three-dimensional flow approximation using the full boundary condition, Equation [5], as described later.

TABLE 3

Measured and Computed Frequencies for a Destroyer

(Frequencies are given in hertz)

Mode	Measured	Computed		
		Strip Method	3-D Flow Using Equation [6]	3-D Flow Using Equation [5]
Heave	--	0.20	0.20	0.20
Pitch	--	0.22	0.23	0.23
First Vib	1.68	1.64	1.64	1.66
Second Vib	3.35	3.16	3.20	3.22
Third Vib	4.97	4.92	4.96	5.00
Fourth Vib	6.63	6.90	6.96	7.03

The results of all three methods are clearly fairly reasonable and there is very little to choose between them. The three-dimensional flow theory using the approximate boundary condition, Equation [6], gives results very similar to the strip method using Lewis correction factors. Since the approximate boundary condition, Equation [6], is equivalent to the Lewis original one, the agreement really is a justification of the use of strip theory. The results for the three-dimensional flow with the more exact boundary condition, Equation [5], were slightly better and reduced the error in the first mode frequency from 2 1/2 to 1 1/4 percent. The strip theory gave the same, improved, result for the first mode frequency when the Taylor rather than the Lewis correction factor was used. However, Taylor does not give values for the reduction factor for the other modes. For the strip theory results, a separate three-dimensional factor had to be applied for each mode.

The differences between the computed and measured frequencies, however, were slightly larger than the differences between the computed values themselves, indicating that the remaining errors were probably due to factors other than the hydrodynamics. There is still some doubt over the ship mass distribution, the material to be included in calculating the stiffness distribution along the ship, and, probably most important, the best method for calculating the shear area distribution along the ship.

The calculated mode shapes were practically identical for the three methods; they are compared with the experimental shapes for the first three vibration modes in Figure 5. The difference between the measured and calculated shapes for the first two modes was less than the scatter in the experimental values, but there was a definite difference between the shapes near the bow for the third mode.

The computer program which produced the results for both the three-dimensional flow approximations is a modification of part of the FORTRAN IV program described in Reference 13 and is run on an IBM 7090 computer.

USE OF THE FULL BOUNDARY EQUATION

Where fairly sharp changes occur in the cross-sectional shape, the three-dimensional flow approximation can be improved by using the full boundary condition (Equation [5]) in place of the approximate condition (Equation [6]). Using the finite element/lumped mass approach, with n masses, the elastic nature of the ship may be represented by the equation

$$\begin{bmatrix} \underset{\sim}{F} \\ \underset{\sim}{Q} \end{bmatrix} = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix} \begin{bmatrix} \underset{\sim}{y} \\ \underset{\sim}{\Upsilon} \end{bmatrix}$$

where $\underset{\sim}{F}$ and $\underset{\sim}{Q}$ are n-component vectors that respectively represent the forces and moments which must be applied to the lumped masses to maintain statically the displacements $\underset{\sim}{y}$ and bending rotations $\underset{\sim}{\Upsilon}$ (Υ excludes shear deformations). A, B, and C will be (n x n) matrixes whose elements are given in Reference 13. In vibration applications, the forces and moments are just the inertial and hydrodynamic forces and moments acting on the regions around the lumped masses. $\underset{\sim}{F}$ and $\underset{\sim}{Q}$ will, therefore, be given by

$$\begin{bmatrix} \underset{\sim}{F} \\ \underset{\sim}{Q} \end{bmatrix} = - \begin{bmatrix} M & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \ddot{\underset{\sim}{y}} \\ \ddot{\underset{\sim}{\Upsilon}} \end{bmatrix} - \begin{bmatrix} M_{w1} & M_{w2} \\ M_{w3} & M_{w4} \end{bmatrix} \begin{bmatrix} \ddot{\underset{\sim}{y}} \\ \ddot{\underset{\sim}{\Upsilon}} \end{bmatrix}$$

where M and R are diagonal matrixes whose elements are the ship masses and rotary inertias at the lumped-mass positions. The matrixes M_{w1}, M_{w2}, M_{w3}, and M_{w4} will be hydrodynamic mass and inertia matrixes.

The above elastic matrices are of order (2n x 2n), and there are several alternatives for the hydrodynamic formulation. The number of sections on which the dipole distributions are defined may be doubled or the same number of distributions may be kept, but the distributions themselves must be given a linear variation instead of being constant. With each technique, either the full or the approximate boundary conditions could be used although the full condition is the more appropriate. To determine the unknown strengths of the 2n dipole distributions, 2n collocation points would be required. As an alternative, the same dipole

distribution can be assumed but be made to satisfy the full boundary condition. It can be used to calculate both the forces and the moments at the lumped masses. This latter technique is in accord with using rotary inertia terms in the elastic representation of the ship instead of doubling the number of elastic sections and lumped masses, and is adopted here.

The vertical velocity at the i^{th} mass is $\dot{y}_i$ and the angular velocity is $\dot{\gamma}_i$. Therefore, substituting for the angle λ, the full boundary condition [5] at the position z_i is

$$\dot{y}_i + b_i b_i' \dot{\gamma}_i = \sum_{j=1}^{n} \left[u_{rj}(z_i) - b_i' u_{zj}(z_i) \right] = \frac{1}{b^2} \sum_{j=1}^{n} \bar{\alpha}_{ij} \mu_j \qquad [14]$$

where

$$\bar{\alpha}_{ij} = \alpha_{ij} + b^2 b_i b_i' \left\{ \left[b_i^{\,2} + (z_i - z_j + \ell_j/2)^2 \right]^{-3/2} - \left[b_i^{\,2} + (z_i - z_j - \ell_j/2)^2 \right]^{-3/2} \right\}$$

Equation [14] may be written in matrix form as

$$\underset{\sim}{\dot{y}} + \bar{B} \underset{\sim}{\dot{\gamma}} = \frac{1}{b^2} \bar{A} \underset{\sim}{\mu} \quad .$$

where $\bar{B}$ is the diagonal matrix with elements $b_i b_i'$. The strengths of the dipole distributions are then

$$\underset{\sim}{\mu} = b^2 \bar{A}^{-1} \underset{\sim}{\dot{y}} + b^2 \bar{A}^{-1} \bar{B} \underset{\sim}{\dot{\gamma}} \qquad [15]$$

Given the strengths of the dipole distributions, from Equation [15], the velocity potential is given by

$$\phi = \sum_{j=1}^{n} \phi_j$$

where ϕ_j is defined in Equation [2]. This leads to a vertical force $f(z)dz$ on a length dz of ship; $f(z)$ is given by

$$f(z) = -\pi\rho \sum_{j=1}^{n} \left\{ \frac{z - z_{j-1/2}}{\left[b^2 + (z-z_{j-1/2})^2\right]^{1/2}} - \frac{z - z_{j+1/2}}{\left[b^2 + (z-z_{j+1/2})^2\right]^{1/2}} \right\} \dot{\mu}_j \qquad [16]$$

The hydrodynamic force and moment on the i^{th} section are therefore

$$F_i = \int_{z_{i-1/2}}^{z_{i+1/2}} f(z)\, dz \quad \text{and} \quad Q_i = \int_{z_{i-1/2}}^{z_{i+1/2}} (z-z_i)\, f(z)\, dz$$

whence

$$F_i = -\pi\rho \sum_{j=1}^{n} c_{ij}\, \dot{\mu}_j \quad , \quad Q_i = -\pi\rho \sum_{j=1}^{n} d_{ij}\, \dot{\mu}_j$$

In these, c_{ij} and d_{ij} are given by

$$c_{ij}/\ell_i = \left(1+b_i'^2\right)^{-1/2} \left(r_{1+}-r_{1-}-r_{2+}+r_{2-}\right) + \frac{x_1-\alpha_1}{\left(1+b_i'^2\right)^{1/2}} \log\left(\frac{\alpha_1+1/2+r_{1+}}{\alpha_1-1/2+r_{1-}}\right) -$$

$$- \frac{x_2-\alpha_2}{\left(1+b_i'^2\right)^{1/2}} \log\left(\frac{\alpha_2+1/2+r_{2+}}{\alpha_2-1/2+r_{2-}}\right)$$

where $x_1 = \dfrac{z_i-z_j+\ell_j/2}{\ell_i}$, $x_2 = \dfrac{z_i-z_j-\ell_j/2}{\ell_i}$; $\alpha_1 = \dfrac{b_i b_i'/\ell_i+x_1}{1+b_i'^2}$; $\beta_1 = \dfrac{b_i/\ell_i-x_1 b_i'}{1+b_i'^2}$

$r_{1+} = \left[(\alpha_1+1/2)^2 + \beta_1^2\right]^{1/2}$; $r_{1-} = \left[(\alpha_1-1/2)^2 + \beta_1^2\right]^{1/2}$, with

similar expressions for α_2, β_2, r_{2+} and r_{2-} ,

$$d_{ij}/\ell_i^2 = \frac{1}{2}\left(1+b_i'^2\right)^{1/2} \left[(\alpha_1+1/2)r_{1+} - (\alpha_1-1/2)r_{1-} - (\alpha_2+1/2)r_{2+} + (\alpha_2-1/2)r_{2-}\right]$$

$$+ \frac{\beta_1^{\,2}}{2\left(1+b_i'^{\,2}\right)^{1/2}} \log\left(\frac{\alpha_1+1/2+r_{1+}}{\alpha_1-1/2+r_{1-}}\right) - \frac{\beta_2^{\,2}}{2\left(1+b_i'^{\,2}\right)^{1/2}} \log\left(\frac{\alpha_2+1/2+r_{2+}}{\alpha_2-1/2+r_{2-}}\right)$$

$$+ \frac{x_1-2\alpha_1}{\left(1+b_i'^{\,2}\right)^{1/2}} (r_{1+} - r_{1-}) - \frac{x_2-2\alpha_2}{\left(1+b_i'^{\,2}\right)^{1/2}} (r_{2+} - r_{2-})$$

$$- \frac{2\alpha_1 + \dfrac{\alpha_1^{\,2}+\beta_1^{\,2}}{x_1-2\alpha_1}}{\left(1+b_i'^{\,2}\right)^{1/2}} \log\left(\frac{\alpha_1 + 1/2 + r_{1+}}{\alpha_2 - 1/2 + r_{1-}}\right) + \frac{2\alpha_2 + \dfrac{\alpha_2^{\,2}+\beta_2^{\,2}}{x_2-2\alpha_2}}{\left(1+b_i'^{\,2}\right)^{1/2}} \log\left(\frac{\alpha_2 + 1/2 + r_{2+}}{\alpha_2 - 1/2 + r_{2-}}\right)$$

With these values and uniform length sections,

$$\begin{bmatrix} \underset{\sim}{F} \\ \\ \underset{\sim}{Q} \end{bmatrix} = - \pi\rho b^2 \begin{bmatrix} \bar{C}\,\bar{A}^{-1} & \bar{C}\,\bar{A}^{-1}\bar{B} \\ \\ \bar{D}\,\bar{A}^{-1} & \bar{D}\,\bar{A}^{-1}\bar{B} \end{bmatrix} \begin{bmatrix} \ddot{\underset{\sim}{y}} \\ \\ \ddot{\underset{\sim}{\gamma}} \end{bmatrix} \qquad [17]$$

where $\bar{C}$ and $\bar{D}$ are the matrixes (c_{ij}) and (d_{ij}), which gives the hydrodynamic matrixes M_{w1}, M_{w2}, M_{w3}, M_{w4}. The equation of ship vibration is then

$$\begin{bmatrix} M + M_{w1} & M_{w2} \\ M_{w3} & R + M_{w4} \end{bmatrix} \begin{bmatrix} \ddot{\underset{\sim}{y}} \\ \ddot{\underset{\sim}{\gamma}} \end{bmatrix} + \begin{bmatrix} A & B \\ B^T & C \end{bmatrix} \begin{bmatrix} \underset{\sim}{y} \\ \underset{\sim}{\gamma} \end{bmatrix} = 0 \qquad [18]$$

Since the rotary inertia corrections are small, the matrixes M_{w3}, R, and M_{w4} will be small. If they are neglected, the equation for $\ddot{\underset{\sim}{\gamma}}$ gives

$$\underset{\sim}{\gamma} = - C^{-1}\, B^T\, \underset{\sim}{y}$$

and the $\ddot{\underset{\sim}{y}}$ equation becomes

$$\left[M+M_{w1} - M_{w2}\, C^{-1}\, B^T\right] \ddot{\underset{\sim}{y}} + \left[A - B\, C^{-1}\, B^T\right] \underset{\sim}{y} = 0 \qquad [19]$$

This equation is similar to Equation [14] but allows fully for the effect of changing cross sections and bending deformation in the boundary condition.

The force distributions given by Equation [16] for the vibrating ellipsoids described earlier have been added to Figures 3 and 4 where these differ appreciably from the earlier results. The great improvement in the accuracy of the results for the higher modes is very marked, particularly for the smaller length/beam ratio ellipsoid. The results with the full boundary condition are everywhere almost identical to the exact values.

In computing these values, the slopes b' of the ellipsoids were estimated numerically from the given radii b at the collocation points in order to reduce the amount of data needed to specify the shape of each ellipsoid to that normal for strip-flow calculations. Since this method is clearly adequate for the rather extreme slopes involved in ellipsoids, it should also be satisfactory for ships where shape changes are less severe. Thus in applications to ship vibration, even the more exact form, Equation [19], of the three-dimensional flow approximation need involve no more data than presently necessary for the usual strip method.

The results given by Equation [19] for ship vibration have also been computed for the destroyer case given earlier. The frequencies computed are given in the last column of Table 3 and show a slight improvement in the predicted first mode frequency. Again, the values of b were estimated numerically from the equivalent radii at the collocation points so that no extra data were required in the calculation. As expected, the improvement resulting from the use of the exact boundary condition, Equation [5], instead of the approximate form, Equation [6], was much less marked for the ship than for the ellipsoid. The mode shapes for the destroyer showed no significant change.

CONCLUSIONS

A method is proposed for approximating the effects of the full three-dimensional flow around a ship undergoing transverse vibration. It provides an alternative to the "strip theory" usually used in the calculation of vibration frequencies.

Although the matrixes involved are slightly more complicated to set up, all vibration frequencies and mode shapes can be found from the eigenvalues and vectors of a single matrix equation. In the strip method it is necessary to consider a different matrix equation for each mode shape. The same data are required for either the three-dimensional approximation or the strip method.

Where unusual mode shapes are involved (e.g., for damaged ships with a very weak section) or for the more extreme shapes of ship (small length to beam ratios), the proposed method will give better results than with strip theory. Otherwise, if separate consideration of each mode is acceptable, strip theory is easier to apply and gives very similar results.

The close agreement between strip theory results and those from the three-dimensional analysis indicates that the remaining discrepancies between the experimental and computed vibration frequencies and shapes are largely attributable to inadequacies in the specification of the elastic stiffness characteristics of ships, such as cross-sectional inertia and shear area, rather than to inadequate representation of the hydrodynamic forces.

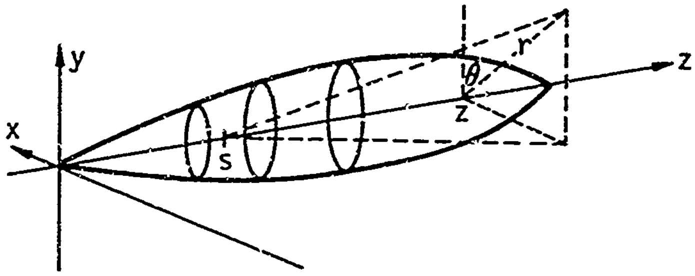

Figure 1 - Geometry and Coordinate System

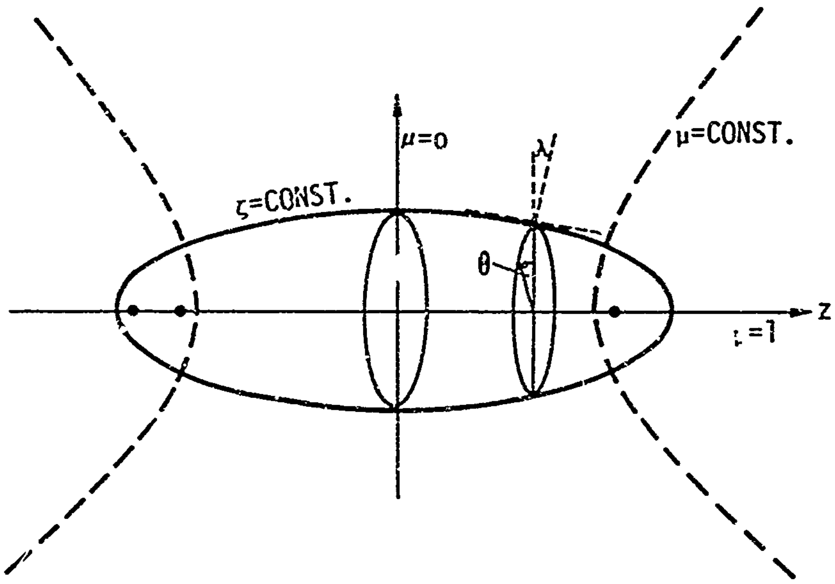

Figure 2 - Coordinate System for
Vibrating Ellipsoid

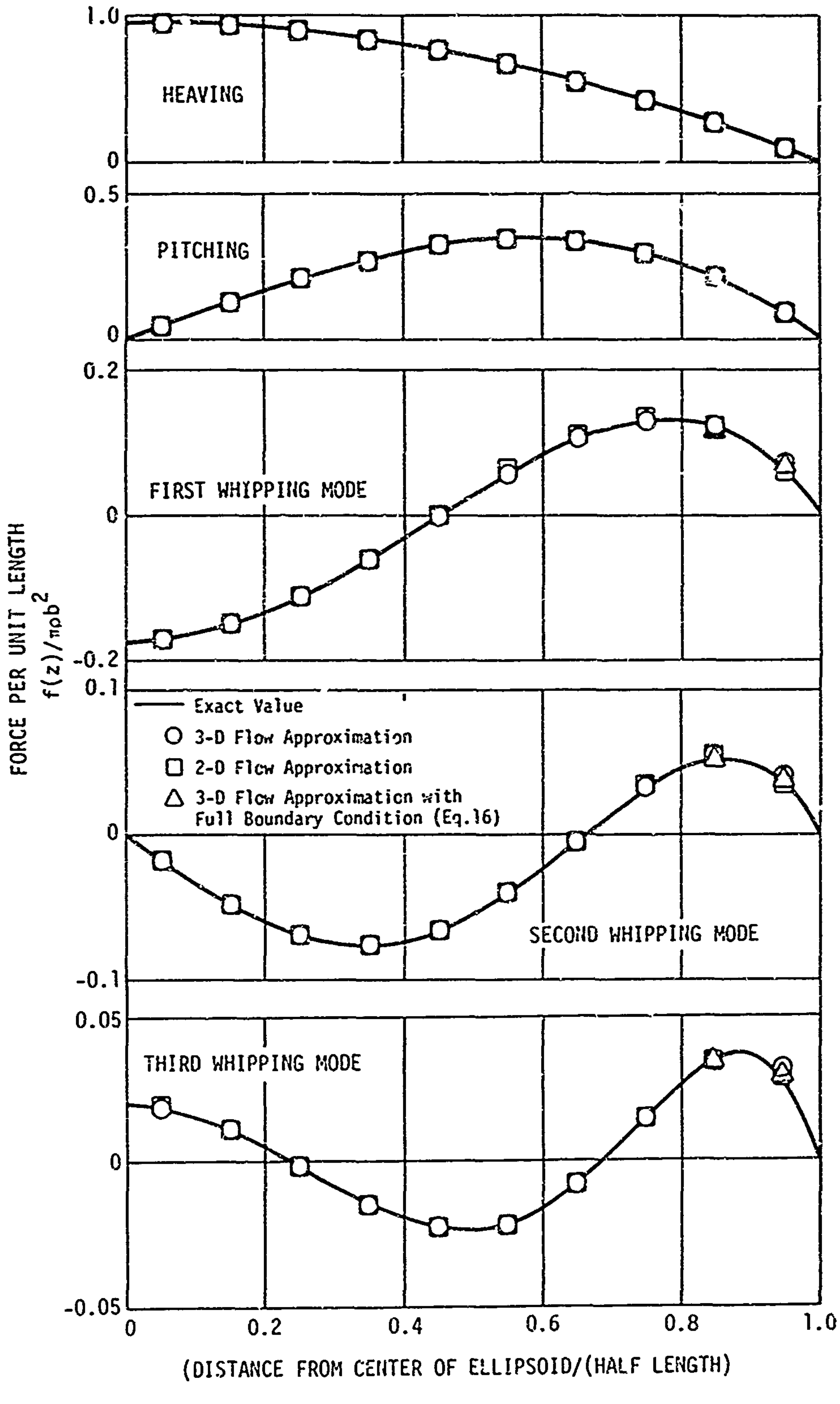

Figure 3 - Comparison of Three- and Two-Dimensional Flow Approximations with the Exact Solution for a Vibrating Ellipsoid (L/B=10)

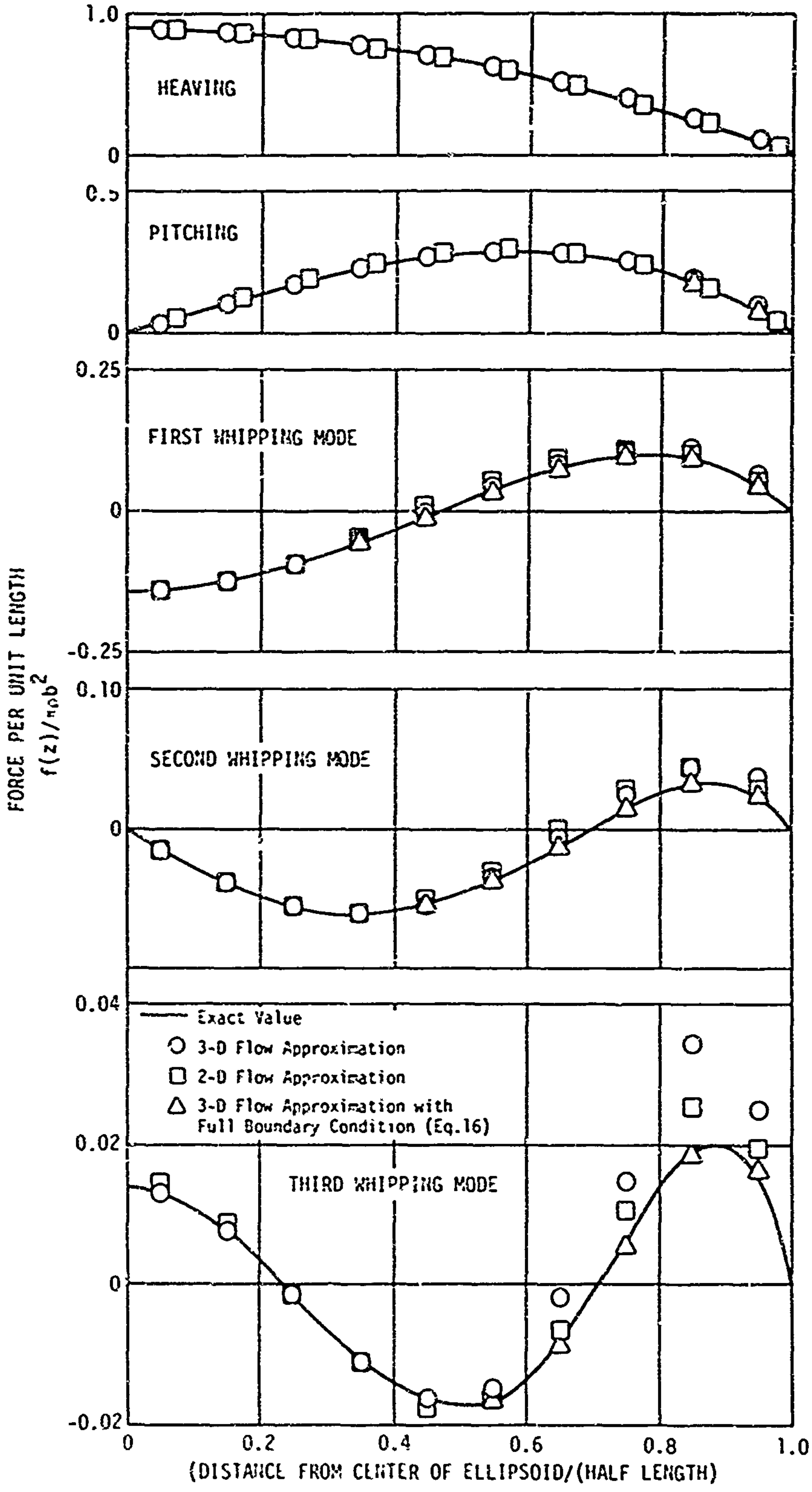

Figure 4 - Comparison of Three- and Two-Dimensional Flow Approximations with the Exact Solution for a Vibrating Ellipsoid (L/B=5)

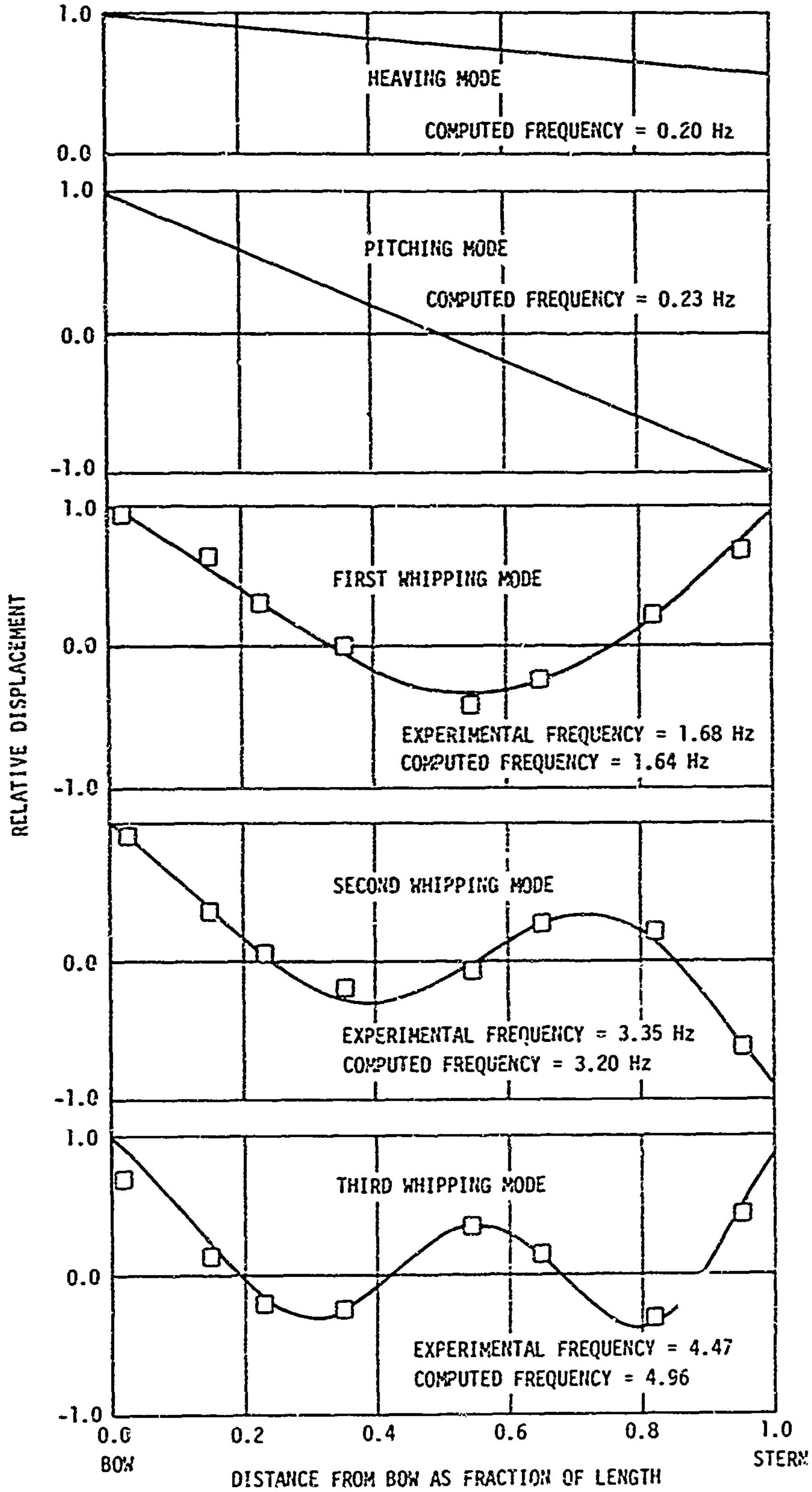

Figure 5 - Measured and Computed Mode Shapes for a Destroyer

APPENDIX

EXACT SOLUTION FOR TRANSVERSE VIBRATION OF AN ELLIPSOID OF REVOLUTION

The coordinate system used to solve this problem is the ellipsoidal set (ζ, μ, θ) illustrated in Figure 2. The curves ζ =const and μ =const represent confocal (and so orthogonal) systems of ellipsoids and hyperboloids of two sheets, respectively. In terms of the cylindrical polar coordinates (r, θ, z),

$$r = k(\zeta^2-1)^{1/2}\,(1-\mu^2)^{1/2} \qquad \text{[A-1]}$$

$$z = k\,\zeta\,\mu$$

If $\zeta = \zeta_o$ represents the given ellipsoid, which is assumed to be of total length 2a and maximum diameter 2b, then $k = (a^2 - b^2)^{1/2}$ and $\zeta_o = (1 - \beta^2)^{-1/2}$ where

$$\beta = b/a \qquad \text{[A-2]}$$

If the vertical velocity distribution along the axis of the ellipsoid is v(z/a), then the boundary condition on the surface of the ellipsoid will be

$$-\left.\frac{\partial\phi}{\partial n}\right|_{\zeta=\zeta_o} = \left[v\cos\lambda - \frac{dv}{dz}\,b\sin\lambda\right]\cos\lambda \qquad \text{[A-3]}$$

which allows for the rotation of cross sections due to bending. If v(z/a) is a polynomial of degree N, then the solution for the velocity potential can be written (see Reference 6) as

$$\phi = \sum_{n=1}^{N+1} a_n\,Q_n^1(\zeta)\,P_n^1(\mu)\cos\theta \qquad \text{[A-4]}$$

where $P_n^1(\mu)$ and $Q_n^1(\zeta)$ are associated Legendre functions of the first and second kinds, respectively.

Substituting this expression for ϕ into Equation [A-3] gives

$$\frac{ak}{b}\,v(\mu) - \frac{bk}{a}\,\mu\,\frac{dv}{d\mu} = -\sum_{n=1}^{N+1} a_n \left.\frac{d\,Q_n^1}{d\zeta}\right|_{\zeta=\zeta_1} \frac{d\,P_n}{d\mu}$$

(since $z=a\mu$ on the ellipse). Since also $v(z/a) = \sum_{n=1}^{N+1} v_n \mu^{n-1}$, the equation becomes

$$\sum_{n=1}^{N+1} \frac{d P_n}{d\mu} \left[- \frac{b}{ak} \left. \frac{d Q_n^1}{d\zeta} \right|_{\zeta=\zeta_o} a_n \right] = \sum_{n=1}^{N+1} \left[1 - (n-1) \beta^2 \right] v_n \mu^{n-1}$$

Putting $\lambda_n = - \frac{b}{ak} \left. \frac{d Q_n^1}{d\zeta} \right|_{\zeta=\zeta_o} \cdot a_n$ and integrating from 0 to μ gives

$$\sum_{n=1}^{N+1} \lambda_n \left[P_n(\mu) - P_n(0) \right] = \sum_{n=1}^{N+1} \frac{1 - (n-1)\beta^2}{n} v_n \mu^2$$

so that

$$\lambda_m = (m + 1/2) \sum_{n=1}^{N+1} \frac{1 - (n-1)\beta^2}{n} \cdot v_n I_{mn} , \quad (m = 1, \ldots, N + 1)$$

where

$$I_{mn} = \int_{-1}^{+1} \mu^n P_m(\mu) d\mu = 0 , \quad n < m \text{ or } (n-m) \text{ odd}$$

$$= 2 \frac{n(n-1) \ldots (n-m+2)}{(n+m+1)(n+m-1) \ldots (n-m+3)} , \quad (n-m) \text{ even}$$

Thus, given the values v_n, the λ_n and hence the a_n are easily found.

With the coefficients a_n known in Equation [A-4], the force distribution $f(z)$ on the ellipsoid is given by

$$f(z) = - 2 \int_0^{\pi} b(z)\, p \cos \theta \, d\theta \quad \text{where} \quad p = \rho \dot{\phi}$$

so that

$$f(z) = - \pi\rho b(z) \sum_{n=1}^{N+1} \dot{a}_n Q_n^1(\zeta_o) P_n^1(z/a)$$

A short computer routine has been written to compute values of $f(z)/\pi\rho b^2$, given b/a and $v_1 \ldots, v_5$, for a series of values of z. For the routine, N is restricted to 4 since this is sufficient to represent the vibration modes of principal interest.

REFERENCES

1. Todd, F.H., "Ship Hull Vibration," Arnold, London (1961).

2. Kaplan, P., "A Study of the Virtual Mass Associated with the Vertical Vibration of Ships in Water," Stevens Inst. Tech., Davidson Lab. Report 734 (1959).

3. Lewis, F.M., "The Inertia of the Water Surrounding a Vibrating Ship," Trans. SNAME, Vol. 37 (1929).

4. Taylor, J.L., "Some Hydrodynamical Inertia Coefficients," Phil. Mag. S.7 Vol. 9, No. 55 (1930).

5. Leibowitz, R.C. and Kennard, E.H., "Theory of Freely Vibrating Nonuniform Beams, Including Methods of Solution and Application to Ships," David Taylor Model Basin Report 1317 (1961).

6. Lamb, H., "Hydrodynamics," Sixth Edition, Cambridge Univ. Press (1932).

7. Macagno, E.O. and Landweber, L., "Irrotational Motion of the Liquid Surrounding a Vibrating Ellipsoid of Revolution," J. Ship Research, Vol. 2, No. 1 (1958).

8. Prohaska, C.W., "Vibrations Verticales du Navire," Bulletin de L" Association Technique Maritime et Aeronautique (1957).

9. Wendel, K., "Hydrodynamic Masses and Hydrodynamic Moments of Inertia," David Taylor Model Basin Translation 260 (1956).

10. Landweber, L. and Macagno, M., "Added Mass of Two-Dimensional Forms Oscillating in a Free Surface," J. Ship Research, Vol. 1 (1957).

11. Landweber, L. and Macagno, M., "Added Masses of a Three-Parameter Family of Two-Dimensional Forms Oscillating in a Free Surface," J. Ship Research, Vol. 2, No. 4 (1959).

12. Macagno, M., "A Comparison of Three Methods for Computing the Added Mass of Ship Sections," J. Ship Research, Vol. 12, No. 4 (1968).

13. Hicks, A.N., "The Elastic Theory of Explosion Induced Whipping-Computer Program Specification," Nav. Const. Res. Est. Report R 550 (1968).

14. Martin, R.S. and Wilkinson, J.H., "Reduction of the Symmetric Eigenproblem $Ax=\lambda Bx$ and Related Problems to Standard Form," Numerische Mathematik, 11, pp. 99-110 (1968).

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R & D

Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY (Corporate author) Naval Ship Research and Development Center Washington, D.C. 20007	2a. REPORT SECURITY CLASSIFICATION Unclassified
	2b. GROUP

3 REPORT TITLE

A METHOD FOR DETERMINING THE VIRTUAL MASS DISTRIBUTION AROUND A VIBRATING SHIP

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final

5 AUTHOR(S) (First name, middle initial, last name)

A.N. Hicks

6. REPORT DATE	7a. TOTAL NO OF PAGES	7b. NO OF REFS
January 1970	34	14

8a. CONTRACT OR GRANT NO	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO	3272
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10 DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11 SUPPLEMENTARY NOTES	12 SPONSORING MILITARY ACTIVITY
	Naval Ordnance Systems Command

13 ABSTRACT

A method is presented for determining the three-dimensional virtual mass distribution associated with the vertical girder vibration of ships. The method was developed for use with a lumped mass/weightless beam ship representation and is based on a set of dipole distributions along the ship axis. It provides a virtual mass matrix with off-diagonal elements and enables all the vibration frequencies and shapes of the ship to be computed from a single matrix equation. The usual method for determining the frequencies and shapes uses a separate mass matrix for each mode. The method is preferable to the standard one for short or unusual ships and mode shapes, or where it is desirable to include all modes in a single equation. However, if separate consideration of each mode is acceptable, the standard technique is simpler for normal ships.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6801

Unclassified

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Hull Vibrations						
Ship Whipping						
Virtual Mass						
Strip Flow						